Dynamical analysis of fractional-order chemostat model

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Appendix

Algorithm of Adams-type Predictor Corrector Method:

\[
\begin{align*}
h &= \frac{T}{N} \\
m &= \left\lfloor \alpha \right\rfloor \\
\text{for } k = 1 \text{ to } N \text{ do begin} \\
\frac{1}{k!} \sum_{j=0}^{k-1} \binom{k-1}{j} x^{j-1} (k-1)^{k-j-1} \\
\text{end}
\end{align*}
\]

\[
\begin{align*}
J[0] &= J[0] \\
\text{for } j = 1 \text{ to } N \text{ do begin} \\
\frac{1}{k!} \sum_{j=0}^{k-1} \binom{k-1}{j} x^{j-1} (k-1)^{k-j-1} \\
J(k) &= \frac{1}{k!} \sum_{j=0}^{k-1} \binom{k-1}{j} x^{j-1} (k-1)^{k-j-1} \\
\end{align*}
\]

end
Proof of Proposition 1: The two roots of the characteristic polynomial are expressed as,

$$\lambda_{\pm} = \frac{-b \pm \sqrt{b^2 - 4c}}{2}.$$

Equation (2.4) is equivalent to the Routh-Hurwitz case if both roots are real or complex conjugate with negative real parts. Otherwise, the roots become

$$\lambda_{\pm} = \frac{-b \pm i \sqrt{4c - b^2}}{2},$$

and get Equation (2.6) if both roots are complex conjugate with positive real parts.

The mathematical explanation of Routh-Hurwitz condition: By substituting values from Table 1, the eigenvalues can be obtained as

$$P(\lambda) = \lambda^2 + b\lambda + c,$$

$$P(\lambda) = \lambda^2 + \frac{2552}{199875}\lambda + \frac{49}{18750}.$$

$$b = \frac{2552}{199875} > 0, \quad c = \frac{49}{18750} > 0.$$

This show that the condition in Eq. (2.5) is satisfied which is $b > 0$ and $c > 0$. 

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